Student Workbook

Applying the Laws of Logarithms

Name: ____________________ Date: ____________ Class: ________

ESSENTIAL QUESTION

How do the product, quotient, and power rules for logarithms mirror the exponent rules — and why does understanding this connection make expanding and condensing logarithmic expressions feel automatic?

Lesson Overview

Logarithm properties let us rewrite complex log expressions into simpler forms — and vice versa. They arise directly from exponent rules: because $\log_b(x)$ is an exponent, every exponent rule has a matching log rule. Mastering the product, quotient, and power rules plus the change-of-base formula unlocks solving exponential and log equations of any base.

$$\log_b(MN) = \log_b(M) + \log_b(N)$$

Product	$\log_b(MN) = \log_b(M) + \log_b(N)$
Quotient	$\log_b(M/N) = \log_b(M) - \log_b(N)$
Power	$\log_b(M^n) = n \cdot \log_b(M)$
Change of Base	$\log_b(x) = \log(x) / \log(b)$

Restrictions & Order of Operations

- All properties require the same base on both sides; arguments M, N must be positive.
- Expand order: quotient rule first → product rule → power rule.
- Condense order (reverse): power rule first → product/quotient rule last.
- NOT valid: $\log(M+N) \neq \log(M)+\log(N)$, and $\log(M-N) \neq \log(M)-\log(N)$. No rule for a sum/difference inside a log.

Worked Examples

EXAMPLE 1

Expand completely: $\log_2(8x^3/y)$

- Quotient rule: $\log_2(8x^3) - \log_2(y)$
- Product rule: $\log_2(8) + \log_2(x^3) - \log_2(y)$
- Power rule + evaluate $\log_2(8)=3$: $3 + 3\cdot\log_2(x) - \log_2(y)$

Answer: $3 + 3\cdot\log_2(x) - \log_2(y)$

EXAMPLE 2

Expand completely: $\ln(\text{sqrt}(x) \cdot y^2 / z^5)$

- $\text{sqrt}(x) = x^{1/2}$. Quotient rule: $\ln(x^{1/2} \cdot y^2) - \ln(z^5)$
- Product rule: $\ln(x^{1/2}) + \ln(y^2) - \ln(z^5)$
- Power rule: $(1/2) \cdot \ln(x) + 2 \cdot \ln(y) - 5 \cdot \ln(z)$

Answer: $(1/2) \cdot \ln(x) + 2 \cdot \ln(y) - 5 \cdot \ln(z)$

EXAMPLE 3

Condense into a single logarithm: $3 \cdot \log(x) + \log(y) - 2 \cdot \log(z)$

- Power rule (reverse): $3 \cdot \log(x) = \log(x^3)$; $2 \cdot \log(z) = \log(z^2)$
- Now: $\log(x^3) + \log(y) - \log(z^2)$
- Product rule: $\log(x^3\ y) - \log(z^2) \rightarrow$ Quotient rule: $\log(x^{3y}/z^2)$

Answer: $\log(x^3\ y / z^2)$

EXAMPLE 4

Condense: $(1/2) \cdot \log_3(x) - 3 \cdot \log_3(y) + \log_3(z)$

- Power rule: $(1/2) \cdot \log_3(x) = \log_3(\text{sqrt}(x))$; $3 \cdot \log_3(y) = \log_3(y^3)$
- Now: $\log_3(\text{sqrt}(x)) - \log_3(y^3) + \log_3(z)$
- Quotient: $\log_3(\text{sqrt}(x)/y^3) \rightarrow$ Product: $\log_3(z \cdot \text{sqrt}(x)/y^3)$

Answer: $\log_3(z \cdot \sqrt{x} / y^3)$

EXAMPLE 5

Use change of base to evaluate $\log_7(50)$ to four decimal places.

- $\log_7(50) = \log(50)/\log(7)$
- $\log(50) = \log(5 \cdot 10) = \log(5) + \log(10) = 0.6990 + 1 = 1.6990$
- $\log(7) \approx 0.8451 \rightarrow \log_7(50) \approx 1.6990/0.8451 \approx 2.0105$

Answer: $\log_7(50) \approx 2.0105$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Expand completely: $\log_3(27x^2/y)$

Hint: Apply quotient rule first, then product rule, then power rule. Evaluate $\log_3(27)$ directly.

2 Expand completely: $\ln(x^4 \cdot \text{sqrt}(y) / z^3)$

Hint: Rewrite $\text{sqrt}(y) = y^{1/2}$. Then apply quotient → product → power rules in order.

3 Condense into a single logarithm: $2 \cdot \log_5(x) - \log_5(y) + 3 \cdot \log_5(z)$

Hint: Apply power rule first to each term, then product/quotient rules to combine.

4 Condense: $(1/3) \cdot \ln(x) + 2 \cdot \ln(y) - \ln(z)$

Hint: Power rule: (1/3)·ln(x)=ln(cube root of x). Then combine with product and quotient rules.

5 Use change of base to evaluate $\log_4(30)$ to four decimal places.

Hint: $\log_4(30)=\log(30)/\log(4)$. Compute each with a calculator, then divide.

Key Vocabulary

Product Rule	$\log_b(MN) = \log_b(M) + \log_b(N)$. Requires same base and positive arguments.
Quotient Rule	$\log_b(M/N) = \log_b(M) - \log_b(N)$.
Power Rule	$\log_b(M^n) = n \cdot \log_b(M)$. An exponent inside a log becomes a coefficient in front.
Change of Base	$\log_b(x) = \log(x)/\log(b) = \ln(x)/\ln(b)$. Converts any log to base 10 or e.
Expanding	Rewriting a single log of a product/quotient/power as a sum/difference of simpler logs.
Condensing	Rewriting a sum or difference of logs as a single logarithm — the reverse of expanding.
Coefficient	The exponent n becomes a multiplier in front of the log after the power rule is applied.
Like Bases	Product/quotient rules require every log in the expression to share the same base.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Expand: $\log_2(x^5 y)$** Multiple Choice

A $5 \cdot \log_2(x) \cdot \log_2(y)$

B $5 \cdot \log_2(x) + \log_2(y)$

C $\log_2(5x) + \log_2(y)$

D $5 \cdot \log_2(xy)$

2 **Condense: log(x) + log(y) − log(z)** Multiple Choice

- **A** log(x + y − z)
- **B** log(xy/z)
- **C** log(x) · log(y/z)
- **D** log(xz/y)

3 **Which expression equals $3 \cdot \log_5(x) - (1/2) \cdot \log_5(y)$?** Multiple Choice

- **A** $\log_5(x^3 - \sqrt{y})$
- **B** $\log_5(3x - y/2)$
- **C** $\log_5(x^3/\sqrt{y})$
- **D** $\log_5(x^3 \cdot \sqrt{y})$

4 **Evaluate $\log_6(36)$ using the power rule.** Multiple Choice

- **A** 1
- **B** 2
- **C** 3
- **D** 6

5 **Change of base for $\log_3(15)$. Which setup is correct?** Multiple Choice

- **A** log(3)/log(15)
- **B** log(15) · log(3)
- **C** log(15)/log(3)
- **D** log(15) − log(3)

Common Mistakes to Avoid

✗ Writing log(M + N) = log(M) + log(N) — splitting a sum inside a log.	✓ The product rule applies to log(M·N), not log(M+N). There is no rule for the log of a sum.
✗ Forgetting to apply the power rule before condensing: 3·log(x)+log(y) = log(3x+y).	✓ First power rule: $3 \cdot \log(x) = \log(x^3)$. Then product rule: $\log(x^3)+\log(y)=\log(x^3\ y)$.
✗ Applying change of base upside-down: $\log_3(15) = \log(3)/\log(15)$.	✓ $\log_b(x) = \log(x)/\log(b)$. The argument goes on top, the base goes on the bottom.
✗ Mixing bases: $\log_2(x) + \log_3(y) = \log_2(xy)$.	✓ Product and quotient rules require the same base — you cannot combine logs with different bases.

Math Tips

★ Every log property mirrors an exponent rule. If you know $b^m \cdot b^n = b^{m+n}$, you already know the product rule.

★ Expand order: quotient rule first → product rule → power rule. Condense in reverse.

★ Change of base is your calculator bridge — always put the argument on top: $\log_b(x)=\log(x)/\log(b)$.

★ There is NO rule for log(M+N) or log(M−N). If you see a sum inside a log, you cannot split it.

★ To verify a condensed answer, expand it back out — if you get the original expression, you're correct.